

Hydraulics

3rd Year civil

First Term (2009 - 2010)

Chapter (   )

2009 - 2010

Zagazig University
Faculty of Engineering
Water & Water Structure Eng. Dept.

3rd Year Civil Engineering
Open Channel Hydraulics
Sheet no. 2 (Uniform flow in open channel)

A. Class work

1- What are the normal water depths if Q=40m^3/sec, S=10cm/km, and n=0.025 for the following sections, a) Rectangular section with 10ms width, b) Triangular section with side slopes 1:1, c)Trapezoidal section with bottom width 10ms and side slopes 2:1, and d) Circular section 10ms diameter.

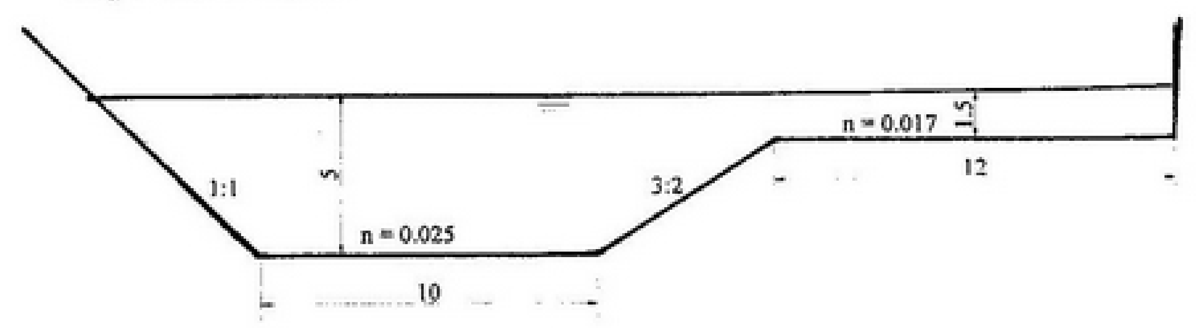
2- A canal with bottom width 20ms, side slopes 2:1, passing a discharge of 300m^3/sec. the normal water depth is 4ms, the canal lined with concrete for which 1/n=80. Determine a) the grade of the canal in cm/km, b) if Q=100m^3/sec, find the corresponding water depth, c) calculate the discharge if S_o is doubled, and d) calculate the discharge if n is doubled.

3- An open channel has a U section, semi-circular at the bottom with vertical side slopes, is 5ft wide. If the normal flow rate is 30c.f.s, the bed slope is 1/4000, and Chezy's coefficient equals 96. Calculate the normal depth and the hydraulic mean depth.

4- An earth channel is lined with concrete (n=0.017), has side slopes 3:2 and is tangent to 4.5ft radius at the bottom, the bed slope is 1/1000 and Chezy's coefficient 100. Calculate the normal water depth, and the hydraulic mean depth.

5- A circular channel conveys 3.25m^3/sec, when 3/4 of the vertical diameter of channel is immersed. The slope of the cannel is 8cm/km. Determine the diameter of the channel (1/n=87.5).

6- For the below compound channel, calculate the total discharge if the bed slope is 1 in 4000.



B. Home work

1- A channel has two sides vertical and semi-circular bottom of 2ms diameter. Calculate the discharge of water through the channel, when the depth of flow is 2ms. Take C=70 and slope of bed as 1 in 1000.

2- A sewer running half full is to be laid at a slope of 1/1000 to serve 200,000 persons at the rate of 300lit/person /day, considering n=0.016, find the sewer diameter if the maximum rate of flow according to which the sewer should be designed can be found by assuming that the total daily discharge flows uniformly in the sewer during 6 hours (not 24 hours).

3- A trapezoidal canal of side slope 1:1 and a bed width four times the depth, conveys $40m^3$/sec, is to be substituted by a semi-circular canal to convey the same discharge at the same velocity. Compare the bed slopes if n=0.012 in both cases.

4- Derive the conditions of the best hydraulic section for the triangular and circular sections.

5- Determine the dimensions of the most economical trapezoidal channel, n=0.016, to carry a discharge of 8000c.f.s with a slope of 12cm/km.

6- A canal having one side vertical and other side is sloping 3:2 caries a discharge of $20m^3$/sec, with a velocity of 0.5m/sec. determine the canal dimensions and its bed slope such that the section is hydraulically best (n=0.025).

7- Show that the maximum discharge in a circular open channel of a certain diameter takes place when the water depth is 0.95 times the channel diameter.

8- A special sewer consists of a semi-circular top and bottom of radius (r) joined by parallel vertical sides of length (2r) so that the total height is (4r), it is required to a) determine the angle subtended by water surface at the center of curvature of the upper semicircle to have maximum discharge, b) if the upper surface is raised until it reached the top of the sewer, find the percentage decrease in the flow.

يقطع هذا الكعب ويسلم مع حل التمرين للمعيد أو المدرس المساعد القائم بتدريس السكشن فى الوقت الذي يحدد له

اسم الطالب: الفصل: الرقم:

7- Derive the conditions of the best hydraulic section for the rectangular and trapezoidal sections.

8- Design the most efficient section of a trapezoidal section of side slopes 2:1, n=0.025, to carry discharge of $60 m^3/sec$. to prevent scour, the maximum allowable velocity is 0.6m/sec. what should be the slope of this channel.

9- Show that the maximum velocity in a circular open channel of a certain diameter take place when the water depth is 0.81 times the channel diameter.

Class Work

Q(1) :

Given:

$Q = 40\ m^3/s$

$S = 10\ cm/km$

$n = 0.025$

Req. :

$y = ??$

a- Rectangular section $b = 10.0m$

b- Triangular section $Z = 1:1$

c- Trapizoidal section

$b = 10\ m \quad , \quad Z = 2:1$

d- Circular section $d = 10\ m$

Sol. : By using Manning eqn:

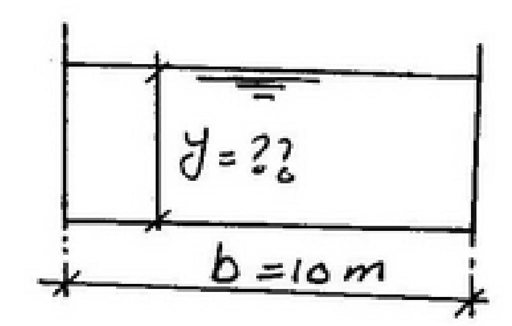
$$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$$40 = \frac{1}{0.025} \times \frac{A^{5/3}}{P^{2/3}} \times (10 \times 10^{-5})^{1/2}$$

$$\therefore \boxed{100 = \frac{A^{5/3}}{P^{2/3}}}$$

Rectangular:

$$\because \quad 100 = \frac{A^{5/3}}{P^{2/3}}$$



$A = 10y$

$P = b + 2y = 10 + 2y$

$$\therefore \quad 100 = \frac{(10y)^{5/3}}{(10+2y)^{2/3}} \quad \text{by trial}$$

y	3	4	5.5	5.30
R.H.S	45.6	68.1	104.5	99.50

$y \simeq 5.35$ m #

Triangular:

$$\because 100 = \frac{A^{5/3}}{P^{2/3}}$$

$T = 2y$, y, y, y, y, 1, 1

$$- A = \frac{1}{2} \times 2y \times y = y^2$$

$$- P = 2\sqrt{y^2 + y^2} = 2\sqrt{2y^2} = 2y\sqrt{2}$$

$$P = 2.83y$$

$$\therefore 100 = \frac{(y^2)^{5/3}}{(2.83y)^{2/3}}$$

$$\therefore 100 = \frac{y^{10/3}}{2 \cdot y^{2/3}} = \frac{y^{8/3}}{2}$$

$$\therefore 200 = y^{8/3}$$

$$y = (200)^{3/8} = 7.30 \text{ m} \quad \#$$

Trapizoidal :

$$\therefore \quad 100 = \frac{A^{5/3}}{P^{2/3}}$$

y = ??

1
2

b = 10 m

$$A = (b + Zy)y = (10 + 2y)y$$

$$P = b + 2y\sqrt{1+Z^2} = 10 + 2y\sqrt{1+Z^2} = 10 + 4.47y$$

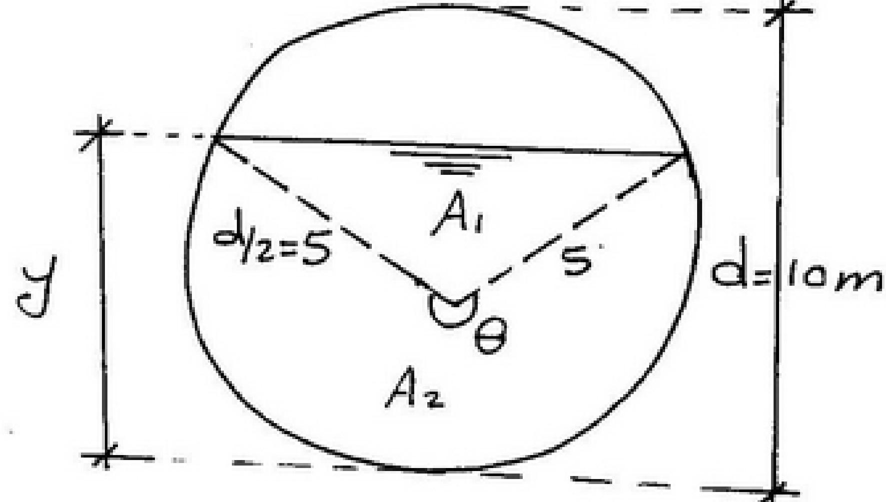
$$\therefore \quad 100 = \frac{\left[(10+2y)y\right]^{5/3}}{\left[10+4.47y\right]^{2/3}} \quad \text{by trial}$$

y	3	4	3.3	3.4	3.45
R.H.S	77.5	135.5	93	98.6	101.4

$y \simeq 3.42$ m #

Circular section:

$\because 100 = \frac{A^{5/3}}{P^{2/3}}$



$\because A_1 = \frac{1}{2} \times (5)^2 \times \sin(360 - \theta)$

$A_1 = -12.5 \sin\theta$

$A_2 = \frac{\pi r^2 \theta}{360} = \frac{\pi \times 5^2 \times \theta}{360}$

$\pi r^2 \rightarrow 360$

$?? \rightarrow \theta$

$A_2 = 0.218\,\theta$

$\rightarrow A = 0.218\,\theta - 12.5 \sin\theta$

$P = \frac{2\pi r \theta}{360} = \frac{2\pi \times 5 \times \theta}{360}$

$2\pi r \rightarrow 360$

$P \rightarrow \theta$

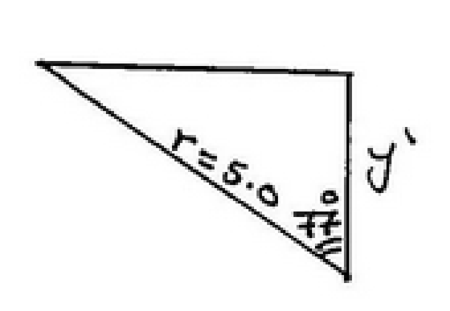
$\rightarrow P = 0.0873\,\theta$

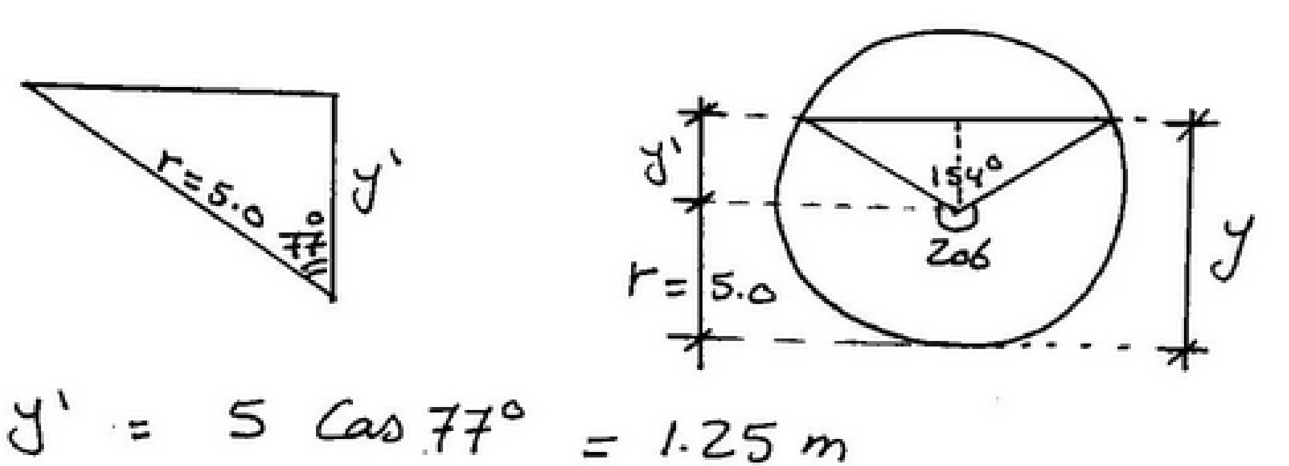
$$\therefore \quad 100 = \frac{\left[0.218\theta - 12.5 \sin\theta\right]^{5/3}}{\left[0.0873\,\theta\right]^{2/3}}$$

by trial

θ	180	200	250	220	214	.....
R.H.S	72.3	93.8	138.8	114	108.4	.....

$$\theta \simeq 206^\circ \quad \#$$





$$y' = 5 \cos 77^\circ = 1.25 \text{ m}$$

$$\therefore y = r + y' = 5 + 1.125$$

$$y = 6.125 \text{ m} \quad \#$$

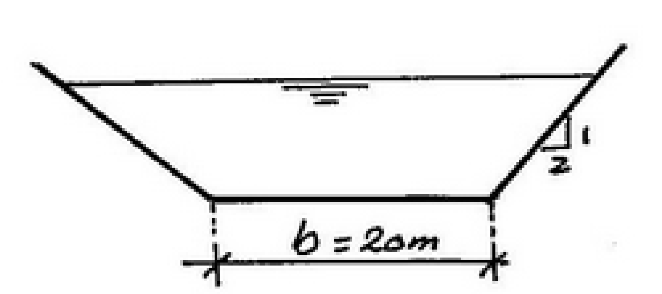
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Class Work

Q(2):

- $Q = 300 \; m^3/s$
- $y = 4.0 m$
- $\frac{1}{n_o} = 80$



Req.:

a- grade of Canal (S_o)

b- $Q = 100 \; m^3/s$ $y = ??$

c- $Q = ??$ $n = 2 n_o$

d- $Q = ??$ $S = 2 S_o$

Sol.:

(a)

$$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$$\because A = (b + Zy)y = (10 + 2 \times 4) \times 4 = 72 m^2$$

$$P = b + 2y\sqrt{1+Z^2} = 10 + 2 \times 4 \sqrt{1 + 2^2} = 27.90 \; m$$

$$\therefore \quad 300 = 80 \times \frac{(72)^{5/3}}{(27.9)^{2/3}} \times S_o^{1/2}$$

$$\therefore \quad S_o = 7.66 \times 10^{-4} \quad \#$$

(b) $\because \; Q = 100 \; m^3/s \quad , \quad b = 20 \; m$

$S = 7.66 \times 10^{-4}$

$\frac{1}{n} = 80 \quad \Rightarrow \quad y = ??$

$$\because \; Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$\because \; A = (10 + 2y)\, y$

$P = 10 + 2y\sqrt{1+2^2} = 10 + 4.47y$

$$\therefore \; 100 = 80 \times \frac{[(10+2y)y]^{5/3}}{[10+4.47y]^{2/3}} \times (7.66 \times 10^{-4})^{1/2}$$

$$\therefore \quad 45.16 = \frac{[(10+2y)y]^{5/3}}{[10+4.47y]^{2/3}}$$

by trial.

y	3	2.5	2.3	2.25
R.H.S	77.5	54.90	46.99	45.12

$$y = 2.25 \text{ m} \quad \#$$

(C) $b = 20 \text{ m}$, $y = 4.0 \text{ m}$, $Z = 2$

$S = 7.66 \times 10^{-4}$

$\because n_o = \frac{1}{80} = 0.0125$

$\therefore n = 2 \times 0.0125 = 0.025$

$\therefore \frac{1}{n} = 40$

$$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$A = 72 \text{ m}^2$, $P = 27.9 \text{ m}$

$$\therefore Q = 40 \times \frac{(72)^{5/3}}{(27.9)^{2/3}} \times (7.66 \times 10^{-4})^{1/2}$$

$$Q = 150 \text{ m}^3/\text{s} \quad \#$$

(d) $b = 20\ m$, $y = 4.0 m$

$2S_o = S = 7.66 \times 10^{-4} \times 2 = 1.53 \times 10^{-3}$

$\frac{1}{n} = 80$, $Z = 2$

$Q = ??$

$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$

$\therefore A = 72.0\ m^2$

$P = 27.90\ m$

$\therefore Q = 80 \times \frac{(72.0)^{5/3}}{(27.9)^{2/3}} \times (1.53 \times 10^{-3})^{1/2}$

$Q = 423.90\ m^3/s$ #

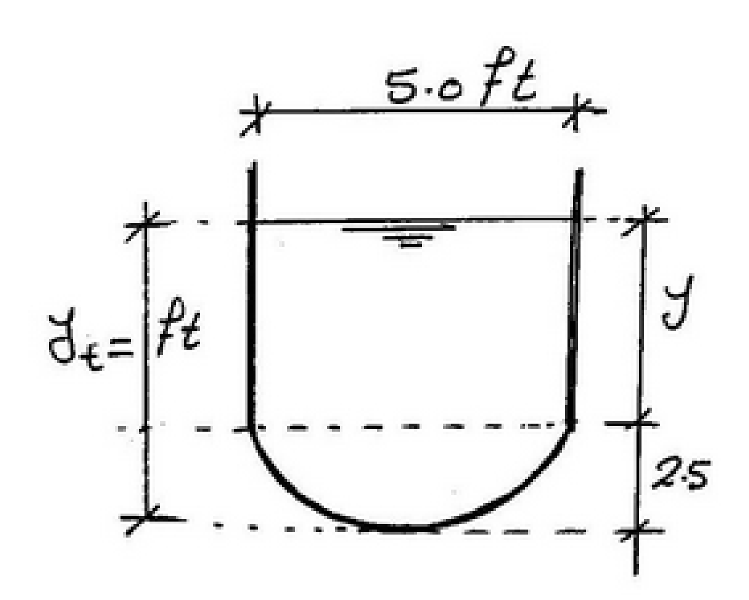
Q (3) :

Given :

$Q = 30 \ ft^3/s$

$S = 1/4000$

$C = 96$



Req. : y_t , y_h

sol. :

$$\because \ Q = C \times \frac{A^{3/2}}{P^{1/2}} \times S^{1/2}$$

$$\because \ A = A_{Rectangle} + A_{semicircle}$$

$$A = 5y + \frac{\pi \times 2.5^2}{2} = 5y + 9.82$$

$$P = 2y + \frac{2\pi \times 2.5}{2} = 2y + 7.90$$

$$\therefore \ 30 = 96 \times \frac{[5y + 9.82]^{3/2}}{[2y + 7.9]^{1/2}} \times (1/4000)^{1/2}$$

$$\therefore \quad 19.76 = \frac{[5y + 9.82]^{3/2}}{[2y + 7.9]^{1/2}} \quad \text{by trial}$$

y	1	1.10	1.30	1.20
R.H.S	18.11	18.9	20.3	19.60

$$y \simeq 1.22 \text{ ft}$$

$$\therefore y_t = 1.22 + 2.5 = 3.72 \text{ ft} \ \#$$

$$\because y_h = \frac{A}{T}$$

$$\therefore A = 5 \times 1.22 + 9.82 = 15.92 \text{ ft}^2$$

$$T = 5 \text{ ft}.$$

$$\therefore y_h = \frac{15.92}{5}$$

$$y_h = 3.18 \text{ ft} \ \#$$

Q(5) :

Given :

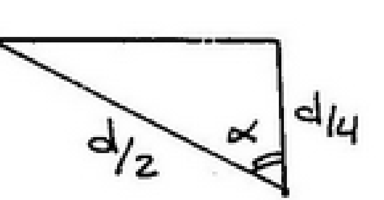
- $Q = 3.25 \ m^3/s$
- $y = \frac{3}{4} d$
- $S = 8 \ cm/km$
- $1/n = 87.5$

Req. : $d = ??$, $y_h = ??$

Sol. :

$$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$\because$



$$\therefore \cos\alpha = \frac{d}{4} \times \frac{2}{d}$$

$$\cos\alpha = \frac{1}{2}$$

$$\therefore \alpha = 60^\circ$$

$$\therefore \theta = 240^\circ$$

$$\because \quad A = \frac{d^2}{8}(\theta_r - \sin\theta)$$

$$= \frac{d^2}{8}\left(\frac{240 \times \pi}{180} - \sin 240\right)$$

$$\leftarrow A = 0.63\, d^2$$

$$\because \quad P = \frac{d}{2}\theta_r = \frac{d}{2} \times \left(\frac{240 \times \pi}{180}\right)$$

$$\leftarrow P = 2.09\, d$$

$$\therefore \quad 3.25 = 87.5 \times \frac{[0.63 d^2]^{5/3}}{[2.09 d]^{2/3}} \times (8 \times 10^{-5})^{1/2}$$

$$4.15 = \frac{0.46\, d^{10/3}}{1.63\, d^{2/3}}$$

$$14.7 = d^{8/3}$$

$$d = (14.7)^{3/8} = 2.75 \text{ m} \ \#$$

$$\because \quad y_h = \frac{A}{T}$$

T, x, d/2 = r = 1.38 m, d/4 = 0.69 m, 60°

$x = 1.38 * \sin 60$

$= 1.20\,m$

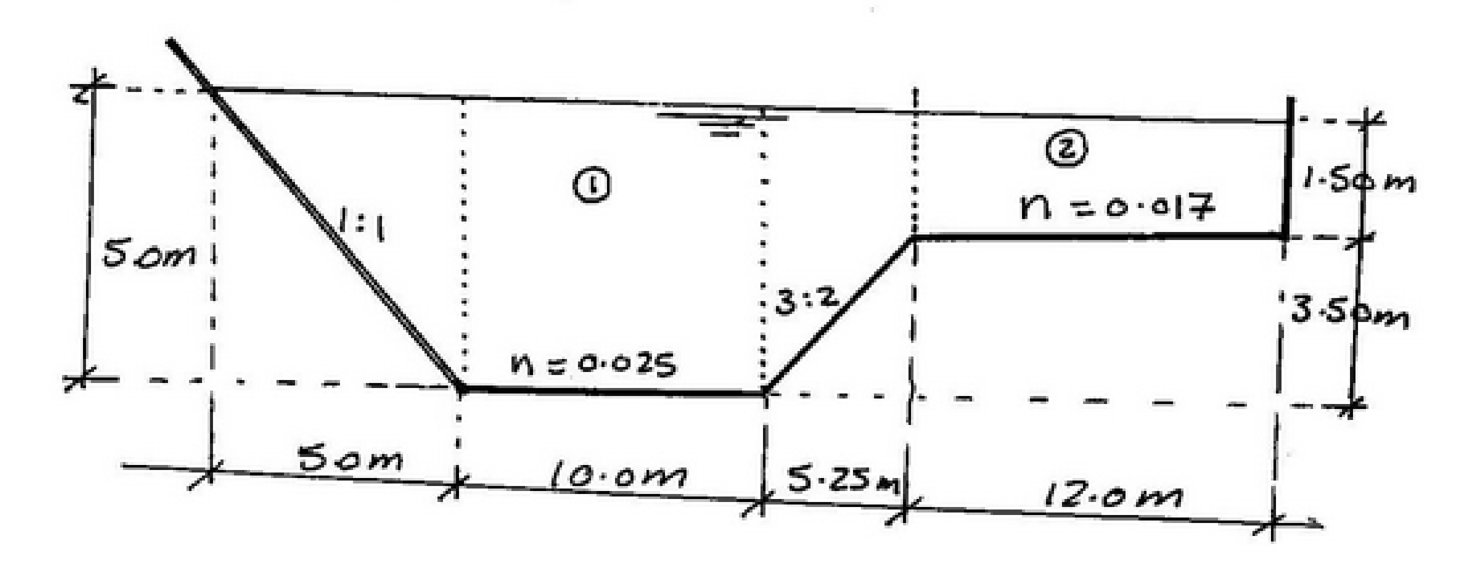
$T = 2x = 2 \times 1.2 =$

$\leftarrow T = 2.40\,m$

$A = 0.63 \times (2.75)^2 = 4.76\,m^2$

$\therefore y_h = \frac{4.76}{2.4} = 1.98\,m$ #

Q(6): Req. = Q = ?? , $S = 1/4000$



method (1) : Dividing Canals

$$\because \quad Q_1 = \frac{1}{n_1} \cdot \frac{A_1^{5/3}}{P_1^{2/3}} \cdot S^{1/2}$$

$$A_1 = (1/2 \times 5 \times 5) + (10 \times 5) + \left[\left(\frac{5+1.5}{2}\right) \times 5.25\right]$$

$$A_1 = 79.60 \ m^2$$

$$P_1 = \sqrt{5^2+5^2} + 10 + \sqrt{5.25^2 + 3.5^2}$$

$$P_1 = 23.38 \ m$$

$$\therefore Q_1 = \frac{1}{0.025} \times \frac{(79.6)^{5/3}}{(23.38)^{2/3}} \times (1/4000)^{1/2}$$

$$Q_1 = 113.94 \ m^3/s$$

$$\because \quad Q_2 = \frac{1}{n_2} \times \frac{A_2^{5/3}}{P_2^{2/3}} \times S^{1/2}$$

$$\therefore A_2 = 12 \times 1.5 = 18 \ m^2$$

$$P = 12 + 1.5 = 13.50 \ m$$

$$\therefore Q_2 = \frac{1}{0.017} \times \frac{(18)^{5/3}}{(13.5)^{2/3}} \times (1/4000)^{1/2}$$

$$Q_2 = 20.28 \ m^3/s$$

$$\therefore Q_t = 113.94 + 20.28 = 134.22 \ m^3/s$$

#

method (2) :

$A_1 = 79.6$

$n_1 = 0.025$

$P_1 = 23.38$

$A_2 = 18$

$n_2 = 0.017$

$P_2 = 13.5$

$$\therefore Q_t = \frac{1}{n_{eq.}} \times \frac{A^{5/3}}{P^{2/3}} \times S^{1/2}$$

$$A = 79.6 + 18 = 97.6 \ m^2$$

$$P = 23.38 + 13.5 = 36.88 \ m$$

$$\therefore \quad n_{eq.} = \left[\frac{\sum P_i \times n_i^{1.5}}{\sum P_i} \right]^{2/3}$$

$$\therefore n_{eq.} = \left[\frac{23.38 * 0.025^{1.5} + 13.5 * 0.017^{1.5}}{23.38 + 13.5} \right]^{2/3}$$

$$n_{eq.} = 0.0222$$

$$\therefore Q_t = \frac{1}{0.0222} \times \frac{(97.6)^{5/3}}{(36.88)^{2/3}} \times (1/4000)^{1/2}$$

$$Q_t = 132.99 \ m^3/s \quad \#$$

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Class Work

Cont.

Q(4):

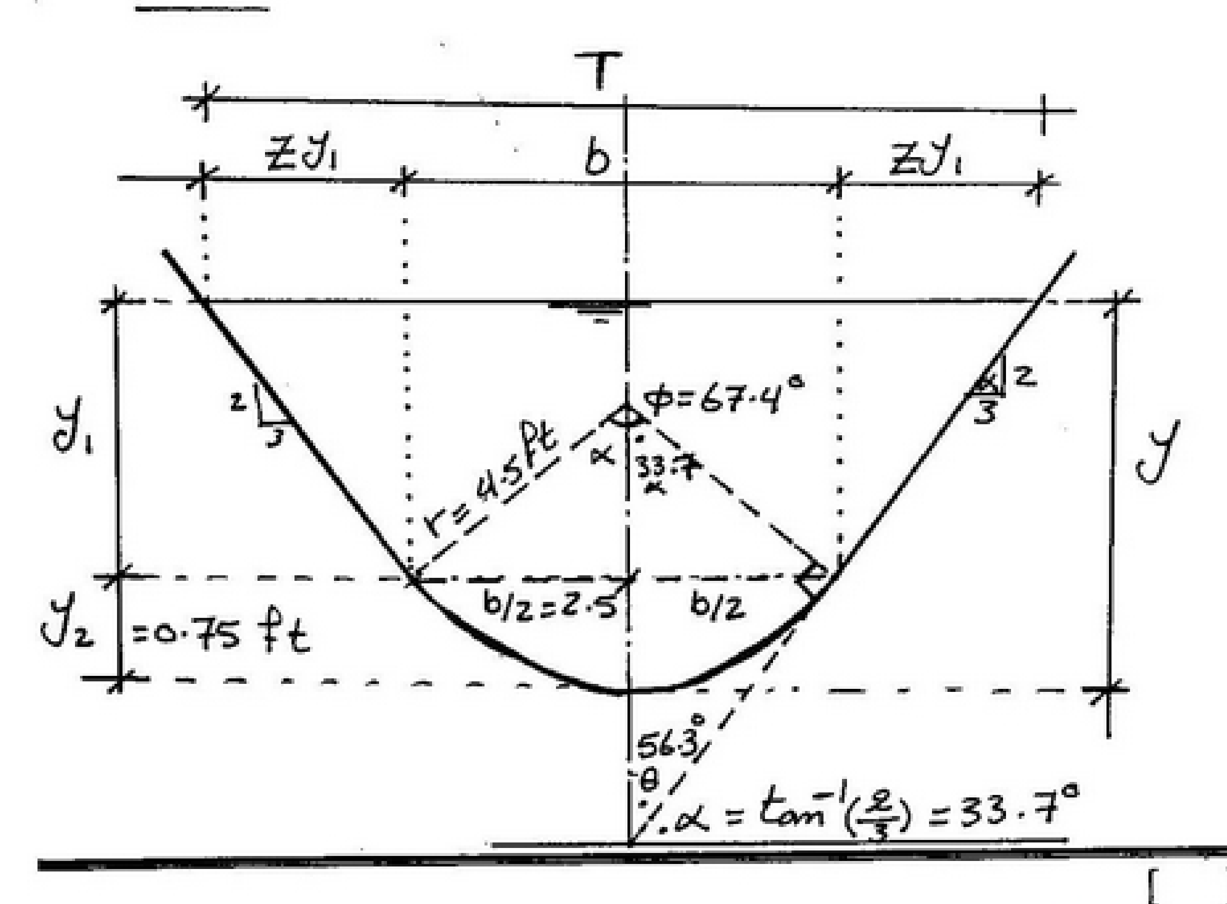
Given: $n = 0.017$, $Z = 3:2$

$S = 1/4000$, $C = 100$

$Q = 300\ ft^3/s$

Req.: $y = ?$ ... $y_h = ?$, $Z = ?$ section factor

Sol.:



$\because Q = \frac{1.486}{n} \times \frac{A^{5/3}}{P^{2/3}} \times S^{1/2}$

$\because A = A_{trap} + A_{circle} - A_{Triangle}$

$\because b/2 = 4.5 \sin 33.7 = 2.5$ ft.

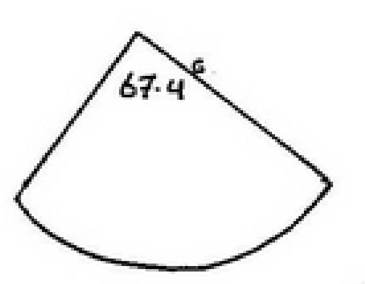
$y_2 = r - 4.5 \cos 33.7$

$= 4.5 - 4.5 \cos 33.7 = 0.75$ ft

$\because A_{Trap} = (b + Zy)y = (5 + 1.5y_1)y_1$

$\because A_{circle} = \frac{\pi \times 4.5^2 \times 67.4}{360}$

$= 11.91$ ft^2



$\because A_{Triangle} = \frac{1}{2} \times 5 \times (4.5 \cos 33.7°)$

$= 9.36$ ft^2

$\pi \times (4.5)^2 \longrightarrow 360°$

$A = ? \longrightarrow 67.4$

$\therefore A = 5y_1 + 1.5y_1^2 + 11.91 - 9.36$

$\leftarrow A = 1.5y_1^2 + 5y_1 + 2.55$

$$\because P = P_{Trap.} + P_{circle}$$

$$\because P_{Trap.} = 2y\sqrt{1+Z^2} = 2y_1\sqrt{1+1.5^2}$$
$$= 3.6\, y_1$$

$$\because P_{circle} = \frac{2\pi \times 4.5 \times 67.4}{360}$$
$$= 5.30 \text{ ft.}$$

$2\pi r \rightarrow 360^\circ$

$P = ? \rightarrow 67.4^\circ$

$$\therefore P = 3.6y_1 + 5.30$$

$$\therefore 300 = \frac{1.486}{0.017} \times \frac{[1.5y_1^2 + 5y_1 + 2.55]^{5/3}}{[3.6y_1 + 5.30]^{2/3}} \times (1/1000)^{1/2}$$

$$\therefore 108.5 = \frac{[1.5y_1^2 + 5y_1 + 2.55]^{5/3}}{[3.6y_1 + 5.3]^{2/3}}$$

y_1	5	4.5	4.6		
R.H.S	127.8	104.2	108.9		

$$y_1 \simeq 4.60 \text{ ft.}$$

$$\therefore y = 4.6 + 0.75 = 5.35 \text{ ft} \#$$

$\because y_h = \frac{A}{T}$

$A = 1.5 \times 4.6^2 + 5 \times 4.6 + 2.55$

$A = 57.3 \ ft^2.$

$T = b + 2Zy = 5 + 2 \times 1.5 \times 4.60$

$= 18.8 \ ft.$

$y_h = \frac{57.3}{18.8} = 3.05 \ ft$ #

$\because Z = A\sqrt{y_h} = 57.3\sqrt{3.05}$

$= 100.10$ #

Q(8)

Given: $Z = 2:1$, $n = 0.025$

$Q = 60\ m^3/s$, $V = 0.6\ m/s$

Req.: - Design most efficient section

$b = ?$, $y = ?$

- slope of Canal $S = ?$

Sol.:

$$\because Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$$

$$\because A = (b + Zy)\, y = (b + 2y)\, y$$

$$P = b + 2y\sqrt{1 + Z^2} = b + 4.47y$$

$$\because Q = A \times V$$

$$60 = A \times 0.6$$

$$\therefore A = 100\ m^2$$

For B.H.S $R = \frac{y}{2} = \frac{A}{P}$

$$\therefore \frac{(b+2y)y}{b+4.47y} = \frac{y}{2}$$

$$2b + 4y = b + 4.47y$$

$$\boxed{b = 0.47y}$$

$$\because \quad 100 = (b + 2y)y$$

$$\therefore \quad 100 = (0.47y + 2y)y$$

$$\therefore \quad 100 = 2.47y^2$$

$$\therefore y = 6.4 \text{ m} \ \#$$

$$b = 3.00 \text{ m} \ \#$$

$$\therefore A = 100 \text{ m}^2$$

$$P = 3 + 4.47 \times 6.4 = 31.6 \text{ m}$$

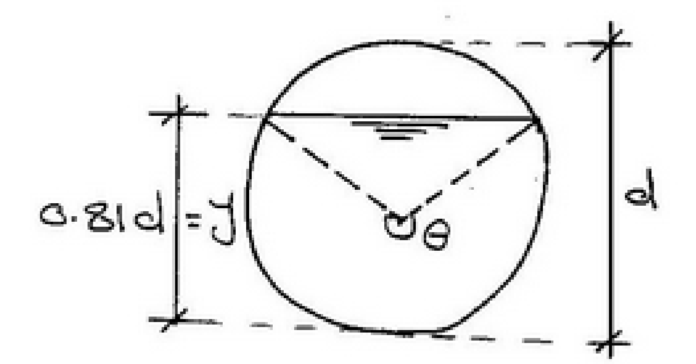
$$\because \ 60 = \frac{1}{0.025} \times \frac{(100)^{5/3}}{(31.6)^{2/3}} \times S^{1/2}$$

$$S = 4.84 \times 10^{-5} \ \#$$

Q(9) :

for max velocity prove that

$y = 0.81\,d$



Sol. :

$\because \; V = C\sqrt{R \cdot S}$

$V = C\sqrt{\left(\frac{A}{P}\right) \cdot S}$

for V_{max} $\frac{d}{d\theta}\left(\frac{A}{P}\right) = 0$

$$\frac{A \times \frac{dP}{d\theta} - P \times \frac{dA}{d\theta}}{P^2} = 0$$

$$\therefore \; A\,\frac{dP}{d\theta} = P\,\frac{dA}{d\theta}$$

$$\because \quad A = \frac{d^2}{8}(\theta_r - \sin\theta) \longrightarrow 1$$

$$\frac{dA}{d\theta} = \frac{d^2}{8}(1 - \cos\theta) \longrightarrow 2$$

$\because \; P = \frac{d}{2}\,\theta r \longrightarrow 3$

$\frac{dP}{d\theta} = \frac{d}{2} \longrightarrow 4$

$\therefore \frac{d^2}{8}(\theta r - \sin\theta) \times \frac{d}{2} = \frac{d}{2}\,\theta r \times \frac{d^3}{8}(1 - \cos\theta)$

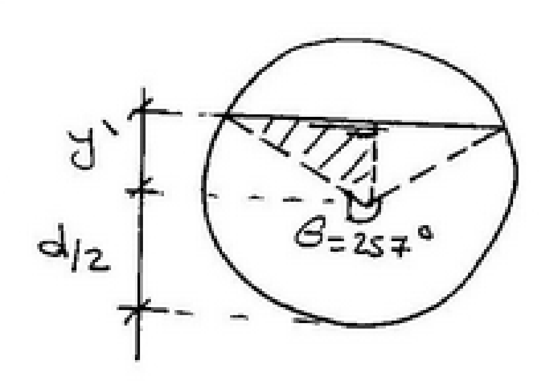
$(\theta r - \sin\theta) = \theta r(1 - \cos\theta)$ by trial

θ	190	200	230	250	255
R.H.S	6.58	6.77	6.59	5.85	5.6
L.H.S	3.48	3.83	4.78	5.30	5.42

$\theta \simeq 257°$

$y' = \frac{d}{2}\cos 51.5$

$= 0.31\,d$



$y = 0.5d + 0.31d$

$y = 0.81\,d$ #

